Ground-state properties and phase/shape transitions in the IBA

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Received: 5 November 2004 / Published online: 3 May 2005 – © Società Italiana di Fisica / Springer-Verlag 2005

Abstract. Detailed fits to energies and electromagnetic transition rates for isotopic chains in the rare-earth region were performed using a simple IBA-1 Hamiltonian. The resulting parameters were then used to calculate two-neutron separation energies, isomer and isotope shifts. Comparison of the isotope shift behavior with other observables in this mass region suggests that the isotope shift could provide an indication for a first-order phase transition.

PACS. 21.10.Re Collective levels – 21.60.Fw Models based on group theory – 27.70. $+q$ 150 $\leq A \leq 189$

The nature of phase/shape transitions as nuclei evolve from spherical to deformed shapes is a fundamental issue and recently has been the focus of many theoretical and experimental investigations. The study of phase transitional behavior in nuclei can easily be accomplished using the Interacting Boson Model (IBA) [\[1\]](#page-1-0), where a study of the total energy surface of the IBA Hamiltonian has shown [\[2\]](#page-1-1) that first- and second-order phase transitions occur as a function of the IBA parameters.

Signatures of phase transitions can be observed in the evolution of observables related to the masses and radii of nuclei. Intuitively, one would expect these quantities to provide the most obvious evidence for phase/shape transitional behavior since they are closely connected to the shape of the nucleus. Observables such as two-neutron separation energies [\[2\]](#page-1-1) and isomer shifts [\[3\]](#page-1-2) have provided experimental evidence of phase transitions.

In order to understand the evolution of these quantities within the framework of the IBA and their connection to actual nuclei, we have performed detailed fits [\[4\]](#page-1-3) to collective even-even nuclei with $Z = 64$ to 72 and $N = 86$ to 104 using the IBA-1 model. Calculations were performed using the extended consistent Q formalism (ECQF) [\[5\]](#page-1-4) with the Hamiltonian $[6,7]$ $[6,7]$

$$
H(\zeta) = c \bigg[(1 - \zeta)\hat{n}_d - \frac{\zeta}{4N_B} \hat{Q}^\chi \cdot \hat{Q}^\chi \bigg]. \tag{1}
$$

The above Hamiltonian contains two parameters, ζ and χ (c is a scaling factor), while N_B is given by half the number of valence protons and neutrons, each taken separately relative to the nearest closed shell.

Parameters for each nucleus were extracted by considering basic properties of the ground, 0^+_2 , and quasi- 2_γ

bands, where the 2^+_γ state is a member of the two-phononlike multiplet in vibrational nuclei or else the bandhead of the quasi- γ band in rotational nuclei. Emphasis was placed on fitting the energy ratios $R_{4/2} \equiv E(4_1^+)/E(2_1^+),$ $E(0_2^+)/E(2_1^+)$, and $E(2_{\gamma}^+)/E(2_1^+)$ as well as the electromagnetic decay of these states. In most cases, a small range of parameter values is able to reproduce the above energy ratios to within 5%. Electromagnetic transition strengths were also reasonably reproduced. The quality of the fits to experimental energies in the Gd and Yb isotopic chains is demonstrated in fig. [1.](#page-1-7) The parameters obtained in the fits to the above spectroscopic information were then used to calculate two-neutron separation energies, isomer and isotopic shifts.

The isomer shift, $\delta \langle r^2 \rangle$, provides a measure of the change in the nuclear radius between the 2^+_1 state and the ground state, given by [\[1\]](#page-1-0)

$$
\delta \langle r^2 \rangle = \langle r^2 \rangle_{2_1^+} - \langle r^2 \rangle_{0_1^+} = \beta \Big[\langle n_d \rangle_{2_1^+} - \langle n_d \rangle_{0_1^+} \Big], \qquad (2)
$$

where the additional parameter β acts only as a scaling factor to connect the results of the calculations to the experimental data.

The isotope shift, $\Delta \langle r^2 \rangle$, provides a measure of the differences in ground-state radii of nuclei differing by one neutron pair, given by [\[1\]](#page-1-0)

$$
\Delta \langle r^2 \rangle^{(N)} = \langle r^2 \rangle_{0_1^+}^{(N+2)} - \langle r^2 \rangle_{0_1^+}^{(N)}
$$

= $\gamma + \beta \Big[\langle n_d \rangle_{0_1^+}^{(N+2)} - \langle n_d \rangle_{0_1^+}^{(N)} \Big],$ (3)

where β is the same quantity as in the isomer shift expression and γ is the contribution from the core which is independent of the structure of the nucleus and the same for the entire region.

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Fig. 1. Comparison of experimental level energies (symbols) and IBA calculations (solid lines) for the 2^+_1 , 4^+_1 members of the ground-state band and the heads of the 2^{+}_{γ} and 0^{+}_{2} bands for the Gd and Yb isotopes.

The results of the calculation for the isomer shift, $\delta\langle r^2\rangle$, for the Gd isotopic chain along with the available experimental data are given in fig. $2(a)$ $2(a)$. The dramatic change in the isomer shift between $N = 88-90$ is reproduced well by the calculations, taking $\beta = 0.03$ fm². The behavior of Gd resembles that of the Sm isotopic chain [\[8\]](#page-1-9), which has been suggested $[3]$ as an indication of a firstorder phase transition.

In fig. $2(b)$ $2(b)$, the results of the calculations for the isotope shift, $\Delta \langle r^2 \rangle$, for Gd are compared to the available experimental data. Again, a sharp change in the isotope shift is observed around $N = 88-90$. Using $\beta = 0.03$ fm² from the isomer shift calculation and $\gamma = 0.15$ fm² gives results which do not reproduce the sharp spike observed at $N = 88$. In order to reproduce the data, a large β value $(= 0.15 \,\mathrm{fm^2})$ and no γ term are necessary. This creates an obvious problem since the IBA parameter β in the isomer and isotope shift is expected to be the same. This is perhaps not surprising since the isomer shift relates to the data in a single nucleus while the isotope shift relates two nuclei differing in neutron number. This suggests that for the isotope shift, IBA-2 calculations might be required. In fact, IBA-2 fits (see for example [\[9\]](#page-1-10)) to isotope and isomer shifts require different values for the proton and neutron components of β .

Fig. 2. Experimental values (symbols) and calculations (lines) of (a) isomer and (b) isotope shifts for the Gd isotopes.

The sharp change in the isotope shift at $N = 88-90$ is evocative of the behavior of both two-neutron separation energies and isomer shifts which also undergo a large change around $N = 88-90$. Since both two-neutron separation energies [\[2\]](#page-1-1) and isomer shifts [\[3\]](#page-1-2) can provide an indication of a first-order phase transition in this mass region, their similarities with the isotope shift behavior is consistent with the concept of a first-order phase transition at $N \sim 90$.

This work was supported by the U.S. Department of Energy under grant DE-FD02-91ER-40609.

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